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Correlations and the Dirac structure of the nucleon self-energy

E. Schiller and H. Müther[a]

Institut für Theoretische Physik, Universität Tübingen, D-72076 Tübingen, Germany

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Abstract. The Dirac structure of the nucleon self-energy in symmetric nuclear matter as well as neutron matter is derived from a realistic meson exchange model for the nucleon-nucleon (NN) interaction. It is demonstrated that the effects of correlations on the effective NN interaction in the nuclear medium can be parameterized by means of an effective meson exchange. This analysis leads to a very intuitive interpretation of correlation effects and also provides an efficient parametrization of an effective interaction to be used in relativistic structure calculations for finite nuclei.

PACS. 21.30.Fe Forces in hadronic systems and effective interactions – 21.65.+f Nuclear matter

1 Introduction

Microscopic studies on the bulk properties of nuclear systems have shown that two ingredients may be needed to derive nuclear properties from realistic nucleon-nucleon interactions: the consideration of correlations beyond the mean-field approach and the relativistic structure of the nucleon self-energy in the nuclear medium. In fact, it is known for many years that attempts to use realistic NN interactions, *i.e.* models for the NN interaction which have been fitted to the NN scattering phase shifts, in a simple mean field or Hartree-Fock calculation lead to nuclear systems which are unbound (see, *e.g.*, the recent review [1]). Therefore, various techniques have been developed to account for correlations beyond the mean-field approach including the Brueckner hole-line expansion [2, 3], the coupled-cluster or "exponential S" approach [4,5], the self-consistent evaluation of Green's functions [6], variational approaches using correlated basis functions [7,8] and recent developments employing quantum Monte Carlo techniques [9,10].

In the framework of the Brueckner theory the effects of two-nucleon correlations are taken into account by evaluating an effective interaction, the so-called G-matrix. This G-matrix corresponds to the T-matrix of NN scattering, however, in the nuclear medium accounting for Pauli and dispersion effects. It is obtained by solving the Bethe-Goldstone equation or in the case of relativistic meson exchange models for the NN interaction by solving an equation which corresponds to a three-dimensional reduction of the Bethe-Salpeter equation like the Blankenbecler-Sugar or the Thompson equation [11]. In the Brueckner-Hartree-Fock (BHF) approximation the nucleon self-energy or single-particle potential is then evaluated in terms of this G-matrix.

Accounting for the two-nucleon correlations in this way or by means of other many-body approaches, one obtains results for the saturation property of nuclear matter or the binding energy and radius of finite nuclei [1], which are quite reasonable. All such results, however, form the so-called Coester band [12], *i.e.* they either predict a binding energy which is too small or a saturation density which is too large (a too small radius in case of finite nuclei) as compared to the empirical values. Extensive studies have been performed to account for three-nucleon correlations [13]. One finds, however, that the inclusion of three-nucleon correlations yields a small effect only, the phenomenon of the Coester band persists for studies within the framework of Brueckner theory as well as within other approaches to solve the many-body problem.

It has been suggested to consider three-nucleon forces, which are adjusted to produce the empirical saturation point of nuclear matter [14]. Another possibility to shift the calculated saturation point away from the Coester band towards the empirical value is to account for relativistic effects. These studies have been motivated by the phenomenology of the Walecka model [15]. The NN interaction in this model is described in terms of the exchange of a scalar meson, σ, and a vector meson ω. Calculating the nucleon self-energy, Σ, from such a meson exchange model within a Hartree approximation, one finds that the ω exchange yields a component Σ^0, which transforms under a Lorentz transformation like the time-like component of a vector, while the scalar meson exchange yields a contribution $\Sigma^{\rm s}$, which transforms like a scalar. Inserting this self-energy into the Dirac equation for a nucleon in the medium of nuclear matter leads to single-particle

[a] e-mail: herbert.muether@uni-tuebingen.de

energies, which are as small as the empirical value of −50 MeV. This small binding effect, however, results from a strong cancellation between the repulsive Σ^0 and the attractive $\Sigma^{\rm s}$ component. The attractive scalar component $\Sigma^{\rm s}$ leads to Dirac spinors for the nucleons in the nuclear medium, which contain a small component significantly enhanced as compared to the Dirac spinor of a free nucleon. This effect is often described in terms of an effective Dirac mass M^* for the nucleon, which can be of the order of 600 MeV in nuclear matter around saturation density. This implies that the Dirac spinors for the nucleons in the nuclear medium are quite different from those in the vacuum. Since these Dirac spinors are used to evaluate the matrix elements for the meson exchange model of the NN interaction, this leads to a medium dependence of this interaction, an effect which influences the calculated saturation property.

Also realistic models for the NN interaction contain large contributions from the exchange of scalar and vector mesons. For such meson exchange potentials V, one can determine the Dirac structure of the nucleon self-energy using the Hartree or Hartree-Fock approximation in a straightforward way. As we have discussed above, using such realistic NN interactions, one has to account for correlations beyond the Hartree-Fock approximation and determine the self-energy in terms of the G-matrix rather than the bare interaction V, *i.e.* perform what is called a Dirac-Brueckner-Hartree-Fock (DBHF) or a relativistic BHF calculation [16–20]. Since, however, this G-matrix is obtained as a solution from a non-relativistic reduction of the scattering equation, it provides matrix elements only between single-particle states and does not keep track of the relativistic structure of the effective interaction.

Various approximation schemes have been developed to determine the Dirac structure of the self-energy Σ or the structure of the nucleon Dirac spinors within the context of the DBHF approach. A rather simple scheme has been suggested by Brockmann and Machleidt [17]. They determine the momentum dependence of the single-particle energy. Identifying this single-particle spectrum with a corresponding spectrum derived within the Dirac-Hartree approximation one can extract the effective Dirac mass M^*. The underlying assumption is that the nucleon self-energy is dominated by the scalar, $\Sigma^{\rm s}$, and time-like vector component, Σ^0, which are constants independent of the momentum of the nucleon. This approximation seems to work reasonably well for symmetric nuclear matter but it fails in the case of neutron matter [21]. Therefore, calculations of the equation of state for asymmetric nuclear matter, which are based on this approach [22,23], should be considered with some caution.

Another scheme, the so-called projection method, analyses the antisymmetrized matrix elements of G in terms of sets of operators, which are invariant under Lorentz transformation. If the relativistic structure of the effective interaction G is defined in this way, one can derive the Dirac structure of the self-energy and determine the density dependence of the nucleon spinors [24,25]. As will be discussed more in detail below, the result of this analysis depends on the choice of relativistic invariants. As an example, we mention the well-known feature that the one-pion exchange contribution yields identical matrix elements for the positive-energy Dirac spinors using pseudo-scalar or pseudo-vector coupling, the Fock contributions to the scalar part of the self-energy, however, are quite different [15,26,27]. As another example, we will discuss the ρ exchange below.

In order to minimise these uncertainties of the projection method, we suggest to split the G-matrix into the Born contribution, the bare interaction V, and the corrections due to the correlations. While the Dirac structure of V is well defined one may employ the projection method for the correction term only. It turns out that these corrections can be described rather well in terms of the exchange of a few effective mesons with high masses, reflecting the short range of the NN correlation effects, and coupling constants depending on the nuclear density. The results of this analysis provides some insight into the effects of NN correlations. This analysis is similar to previous attempts by Boersma and Malfliet [25] and Elsenhans *et al.* [28]. Because of its simple structure, however, it might be more appropriate to be used in DBHF studies on finite nuclei [29,30].

After this introduction a brief description of the main ingredients of the projection technique will be presented in sect. 2. Results of the analysis for the effective NN interaction in symmetric nuclear matter and neutron matter derived from various models of the Bonn potential [11]. After a detailed discussion of the results in sect. 3, the main conclusions will be summarised in sect. 4.

2 Relativistic structure of the G-matrix

The Dirac equation for a nucleon with momentum k in a medium of nuclear matter can be written

$$(\not{k} - M - \Sigma(k))u(\mathbf{k}, s) = 0\,, \tag{1}$$

with the self-energy $\Sigma(k)$ accounting for the mean field generated by the nuclear medium. By the requirement of translational and rotational invariance, parity conservation and time reversal invariance, the general form of the Dirac structure of the self-energy is given in the nuclear matter rest frame as 4×4-matrix by

$$\Sigma(k) = \Sigma^{\rm s}(k) - \gamma^0\Sigma^0(k) + \gamma \cdot \mathbf{k}\Sigma^{\rm v}(k)\,, \tag{2}$$

where $\Sigma^{\rm s}, \Sigma^0, \Sigma^{\rm v}$ are functions, depending for on-shell nucleons ($k^0 = E(\mathbf{k})$) only on the absolute value of the three-momentum $k \equiv |\mathbf{k}|$ and the Fermi momentum $k_{\rm F}$, which is related to the density via $\rho = \delta/(3\pi^2)k_{\rm F}^3$, where the isospin degeneracy yields $\delta = 2$ for nuclear matter and $\delta = 1$ for neutron matter. The density dependence will be suppressed throughout this section.

The components of the self-energy are easily determined by taking traces of the form

$$\begin{aligned}\Sigma^{\rm s}(k) &= \frac{1}{4}{\rm Tr}[\Sigma(k)]\,,\\ \Sigma^{0}(k) &= -\frac{1}{4}{\rm Tr}[\gamma^0\Sigma(k)]\,,\\ \Sigma^{\rm v}(k) &= -\frac{1}{4}{\rm Tr}[\gamma\cdot\hat{k}\Sigma(k)]\,. \end{aligned} \tag{3}$$

Introducing the effective quantities

$$\begin{aligned}M^*(k) &= \frac{M+\Sigma^{\rm s}(k)}{1+\Sigma^{\rm v}(k)}\,,\\ E^*(k) &= \frac{E+\Sigma^{0}(k)}{1+\Sigma^{\rm v}(k)}\,, \end{aligned} \tag{4}$$

the Dirac equation can be rewritten in compact form setting $\mathbf{k}^*=\mathbf{k}$

$$(\not{k}^* - M^*)u(\mathbf{k},s) = 0\,, \tag{5}$$

which is formal identical to the Dirac equation in the vacuum case. Therefore, the positive-energy solution to eq. (5) is given as

$$u(\mathbf{k},s) = \sqrt{\frac{E^*(k)+M^*(k)}{2M^*(k)}}\begin{pmatrix}1\\ \frac{\sigma\cdot\mathbf{k}}{(E^*(k)+M^*(k))}\end{pmatrix}\chi_s \tag{6}$$

with the covariant normalisation

$$\overline{u}u = 1\,, \qquad\qquad u^+u = \tfrac{E^*(k)}{M^*(k)} \tag{7}$$

and the in medium on-shell relation

$$E^{*2}(k) = M^{*2}(k) + \mathbf{k}^2\,. \tag{8}$$

The self-energy itself is connected to the two-particle effective interaction in the medium, the G-matrix, through the Hartree-Fock relation

$$\begin{aligned}\Sigma_{\alpha\beta}(k) = -i\int_{\mathbf{q}\le k_{\rm F}}\frac{{\rm d}^4q}{(2\pi)^4}\tilde{g}_{\tau\sigma}(q)\,\{&G(k,q;k,q)_{\alpha\sigma;\beta\tau}\\ &-G(k,q;q,k)_{\alpha\sigma;\tau\beta}\} =\\ -i\int_{\mathbf{q}\le k_{\rm F}}\frac{{\rm d}^4q}{(2\pi)^4}\tilde{g}_{\tau\sigma}(q)&G^A(k,q;k,q)_{\alpha\sigma;\beta\tau} =\\ -i\int_{\mathbf{q}\le k_{\rm F}}\frac{{\rm d}^4q}{(2\pi)^4}&{\rm Tr}[\tilde{g}(q)G^A(k,q;k,q)]_{\alpha\beta} \end{aligned} \tag{9}$$

with the propagator for real nucleons propagating on-shell inside the Fermi sea in the nuclear matter rest frame

$$\tilde{g}_{\tau\sigma}(q) = \frac{i\pi}{E^*(q)}(\not{q}^*+M^*)_{\tau\sigma}\delta(q^0-E(q))\Theta(k_{\rm F}-|\mathbf{q}|). \tag{10}$$

The equation above (9) must be understood as an operator equation. Therefore, also the G-matrix is needed in terms of Dirac operators as will be discussed now.

One of the main ingredients of the DBHF approach is the derivation of the effective interaction G in the medium from a realistic nucleon-nucleon interaction V. This is achieved by solving the relativistic Bethe-Goldstone equation, a kind of Thomson equation for the scattering of two nucleons in nuclear matter

$$\begin{aligned}G(\mathbf{q}',\mathbf{q}|\mathbf{P},\omega) &= V(\mathbf{q}',\mathbf{q})\\ &+\int\frac{{\rm d}^3k}{(2\pi)^3}V(\mathbf{q}',\mathbf{k})\frac{M^{*2}}{E^{*2}_{\mathbf{P}+\mathbf{k}}}\frac{Q(\mathbf{k},\mathbf{P})}{\omega-2E^*_{\mathbf{P}+\mathbf{k}}}G(\mathbf{k},\mathbf{q}|\mathbf{P},\omega) \end{aligned} \tag{11}$$

with the starting energy

$$\omega = 2E^*_{\mathbf{P}+\mathbf{q}}\,, \tag{12}$$

where $\mathbf{P}=\frac{1}{2}(\mathbf{p}_1+\mathbf{p}_2)$ denotes the center-of-mass momentum and $\mathbf{q}=\frac{1}{2}(\mathbf{p}_1-\mathbf{p}_2)$ is the relative momentum of the initial state, $\mathbf{k}$ and $\mathbf{q}'$ are the relative momenta of the intermediate and final states of two interacting nucleons. The accessible intermediate states are restricted by the Pauli operator Q to momenta above the Fermi momentum $k_{\rm F}$. To shorten the notation isospin indices will be suppressed throughout this section.

To account for the spin, the $\langle\lambda_1'\lambda_2'|G(\mathbf{q}',\mathbf{q})|\lambda_1\lambda_2\rangle$ G-matrix is conveniently obtained in a basis of helicity states, where λ_1',λ_2' and λ_1,λ_2 are the helicities of the final and initial state of particle 1 and 2. Demanding parity conservation, time reversal invariance and conservation of total spin reduces the 16 possible helicity amplitudes to six independent amplitudes, which are normally chosen to be

$$\begin{aligned}&G_1 = \langle++|G(\mathbf{q}',\mathbf{q})|++\rangle\,,\ G_2 = \langle++|G(\mathbf{q}',\mathbf{q})|--\rangle\,,\\ &G_3 = \langle+-|G(\mathbf{q}',\mathbf{q})|+-\rangle\,,\ G_4 = \langle+-|G(\mathbf{q}',\mathbf{q})|-+\rangle\,,\\ &G_5 = \langle++|G(\mathbf{q}',\mathbf{q})|+-\rangle\,,\ G_6 = \langle+-|G(\mathbf{q}',\mathbf{q})|++\rangle\,, \end{aligned} \tag{13}$$

where $+$, $-$ denotes the helicity $\frac{1}{2}$ and $-\frac{1}{2}$, respectively. The explicit dependence on $\mathbf{P}$ and ω has been omitted. The number of independent helicity matrix elements is further reduced to five in the case of on-shell scattering, $|\mathbf{q}'|=|\mathbf{q}|$, which implies

$$G_5 = -G_6\,. \tag{14}$$

In order to evaluate the nucleon self-energy eq. (9), the operator structure of the G-matrix is needed for the on-shell case in the nuclear matter rest frame; on the other hand, the determination of the operator structure is most easily done in the center-of-mass frame, where $\mathbf{P}=0$ and the scattering angle ϑ between the relative momenta $\mathbf{q}'$, $\mathbf{q}$ is fixed at $\vartheta=0$. Therefore, only $q\equiv|\mathbf{q}|$ have to be considered in the following as argument. The operator structure is then given by expanding the G-matrix in terms of five independent Fermi covariants according to the five helicity matrix elements of the G-matrix

$$\begin{aligned}&\langle\lambda_1',\lambda_2'|G(q)|\lambda_1,\lambda_2\rangle_{\rm A} =\\ &\sum_{i={\rm s,v,t,a,ps}}\Gamma^i_{\rm D}(q)\ \langle\ |\hat{T}_i^{(1)}\hat{T}_i^{(2)}|\ \rangle_{\rm D} - \Gamma^i_{\rm X}(q)\ \langle\ |\hat{T}_i^{(1)}\hat{T}_i^{(2)}|\ \rangle_{\rm X} =\\ &\sum_{i={\rm s,v,t,a,ps}}\Gamma^i_{\rm A}(q)\ \langle\lambda_1',\lambda_2',\mathbf{q}|\hat{T}_i^{(1)}\hat{T}_i^{(2)}|\ \mathbf{q},\lambda_1,\lambda_2\rangle_{\rm D} \end{aligned} \tag{15}$$

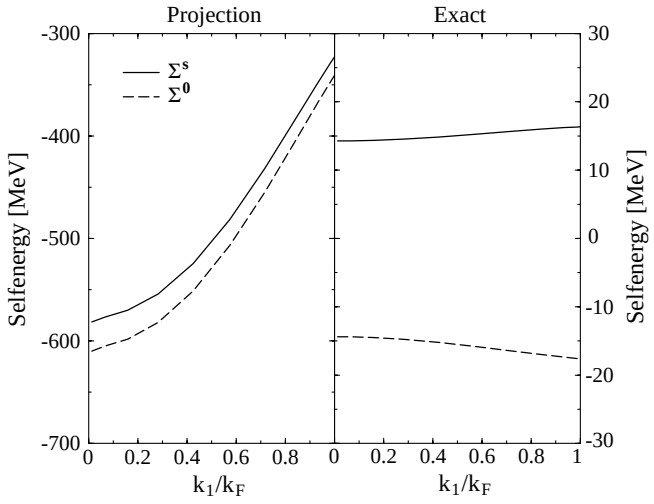

Fig. 1. Contribution of the π exchange to the scalar ($\Sigma^{\rm s}$) and vector component (Σ^0) of the nucleon self-energy in nuclear matter with $k_{\rm F} = 1.36$ fm^{-1}. Results derived from the projection method (pseudo-scalar coupling) are displayed in the left part of the figure, while those derived from the original Bonn interaction (pseudo-vector coupling) are given in the right part. Note the different scales in the two parts of the figure.

with the Fermi covariants

$$\hat{T}_i \in \{1, \gamma^\mu, \sigma^{\mu\nu}, \gamma^5\gamma^\mu, \gamma^5\} \tag{16}$$

and the Lorentz invariant amplitudes

$$\Gamma^i_{\rm A}(q) = \Gamma^i_{\rm D}(q) - \sum_{k=1}^{5} F_{ki}\Gamma^k_{\rm X}(q)\,, \tag{17}$$

where the F_{ki} is the well-known Fierz transformation

$$(F)_{ki} = \frac{1}{4}\begin{pmatrix} 1 & -1 & -\frac{1}{2} & 1 & -1 \\ -4 & -2 & 0 & -2 & -4 \\ -12 & 0 & -2 & 0 & 12 \\ 4 & -2 & 0 & -2 & 4 \\ -1 & -1 & \frac{1}{2} & 1 & 1 \end{pmatrix}. \tag{18}$$

The subscript A in eq. (15) indicates that only antisymmetrized matrix elements are obtained as solution of the Bethe-Goldstone equation and therefore only those can be analysed. Here, the explicit splitting of the antisymmetrized matrix element in its direct and exchange part (labelled with the subscripts D and X) only illustrates, that, using the local Fermi covariants eq. (16), both parts can be rewritten with the help of the Fierz transformation in terms of the antisymmetrized amplitudes $\Gamma^i_{\rm A}$ and the direct matrix element of the Fermi covariants. Nevertheless, the direct and exchange part cannot be determined from the antisymmetrized matrix elements and it is not required to evaluate the self-energy.

Inverting eq. (15) yields the antisymmetrized amplitudes $\Gamma^i_{\rm A}$, where only the scalar $\Gamma^{\rm s}_{\rm A}$ and vector $\Gamma^{\rm v}_{\rm A}$ amplitudes are needed to evaluate the self-energy using eq. (9):

$$\Sigma^{\rm s}(k_1) = -\frac{1}{2\pi^2}\int_0^{k_{\rm F}} {\rm d}^3\mathbf{k_2}\ \frac{M^*}{E^*}\ \Gamma^{\rm s}_{\rm A}(q)\,,$$

$$\Sigma^0(k_1) = -\frac{1}{2\pi^2}\int_0^{k_{\rm F}} {\rm d}^3\mathbf{k_2}\ \Gamma^{\rm v}_{\rm A}(q)\,,$$

$$\Sigma^{\rm v}(k_1) = -\frac{1}{2\pi^2}\int_0^{k_{\rm F}} {\rm d}^3\mathbf{k_2}\ \frac{{k_2}^*}{E^*}\ \Gamma^{\rm v}_{\rm A}(q)\,. \tag{19}$$

Here, k_1 and k_2 are the single particle momenta in the nuclear matter rest frame. The relative momentum q, defined in the c.m. frame, is related with k_1 and k_2 via $q =$

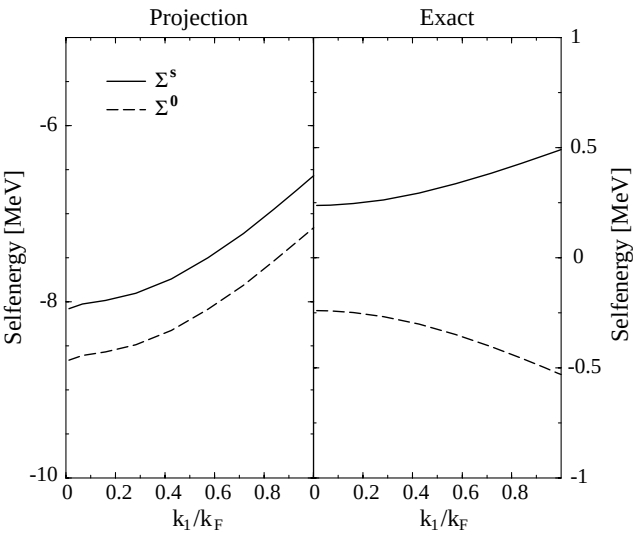

Fig. 2. Contribution of the η exchange to the scalar and vector component of the nucleon self-energy in nuclear matter. For further details, see fig. 1.

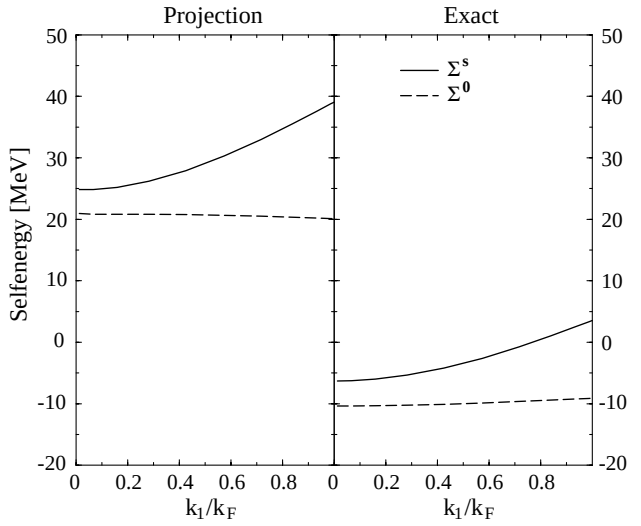

Fig. 3. Contribution of the ρ exchange to the scalar and vector component of the nucleon self-energy in nuclear matter. For further details, see fig. 1.

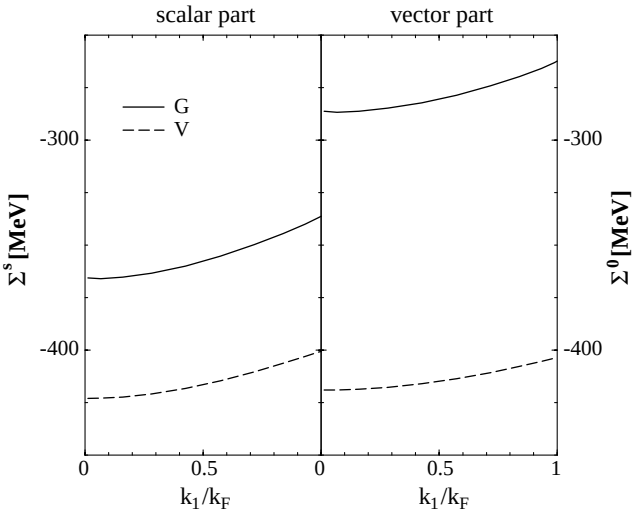

Fig. 4. The scalar ($\Sigma^{\rm s}$, left part) and time-like vector component (Σ^0, right part) of the nucleon self-energy in nuclear matter at $k_{\rm F} = 1.36$ fm^{-1}. Results obtained for the bare NN interaction V (Bonn B potential of [11], dashed lines) are compared to those derived from the G-matrix (solid lines).

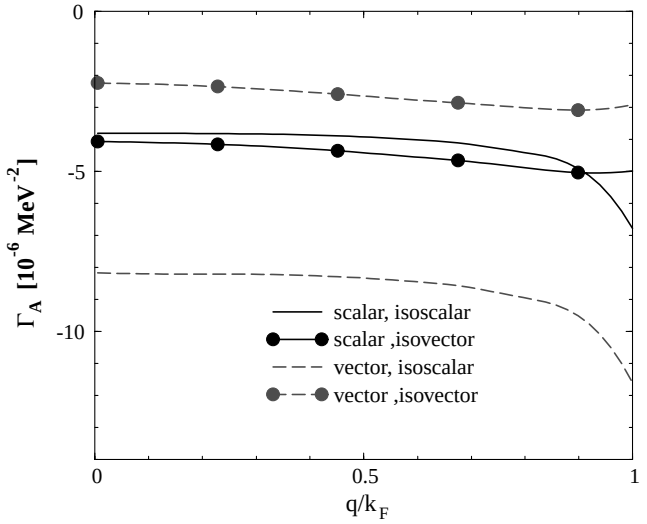

Fig. 5. Antisymmetrized interaction amplitudes $\Gamma^i_{\rm A}$, derived from the analysis of ΔG presented as a function of the relative momentum of the interacting nucleons in their center-of-mass frame.

$\sqrt{s/4 - M^{*2}}$, where $s = (E^*(k_1) + E^*(k_2))^2 - (\mathbf{k_1} + \mathbf{k_2})^2$ is the invariant mass. This transformation between the nuclear matter rest frame and the center-of-mass frame is described in detail in [24].

The used set of Fermi covariants is sufficient to reproduce the matrix elements of the effective interaction G for the positive-energy solutions of the Dirac equations. From the resulting amplitudes $\Gamma^{\rm s}_{\rm A}(q)$ and $\Gamma^{\rm v}_{\rm A}(q)$ one can immediately determine the components of the self-energy according to (19). Nevertheless, this procedure is not unique and depends on the chosen set of operators $\hat{T}$. A well-known example which illustrates this dependence is the pion exchange part of the NN interaction, which either is described by a pseudo-scalar or a pseudo-vector coupling. Both couplings yield for on-shell nucleons the same matrix elements, if the coupling constants $f_{\rm pv}$ and $g_{\rm ps}$ obey the relation $f_{\rm pv}/m_\pi = g_{\rm ps}/(2M)$, where m_π is the pion mass and M the mass of the nucleon. Nevertheless, the components of the self-energy are completely different for both couplings. A more detailed discussion of this and other examples will be given in the next section.

To circumvent the aforementioned problem, which already occurs in the case if the effective interaction G is replaced with the bare nucleon-nucleon interaction V, the G-matrix is split into two parts

$$G = V + \triangle G\,. \tag{20}$$

The decomposition eq. (15) is applied only to the residual part $\triangle G$ since the explicit Dirac structure of V and the way to evaluate the components of the self-energy from V is known explicitly.

3 Results and discussion

In the first part of this section we would like to show some ambiguities of the projection method which is related to the choice of the covariant operators in (16). For that purpose, we consider various components of the bare NN interaction V using as an example the OBE potential Bonn B as defined in table A.2 of [11]. If one considers only the π exchange contribution and analyses the matrix elements of V_π by means of the projection formalism discussed in sect. 2, one obtains large components for Lorentz invariant scalar and vector amplitudes $\Gamma^{\rm s}_{\rm A}$ and $\Gamma^{\rm v}_{\rm A}$ which are due to the Fock-exchange terms of the pseudo-scalar operator representing the π exchange in the projection formalism. Integrating these amplitudes as indicated in (19) leads to large scalar ($\Sigma^{\rm s}$) and time-like vector components (Σ^0).

This is visualised in the left part of fig. 1, where these components calculated for symmetric nuclear matter at the empirical saturation density (Fermi momentum $k_{\rm F} = 1.36$ fm^{-1}) are presented as a function of the nucleon momentum k_1. Note that this analysis yields an absolute value for the scalar component which gets as large as 600 MeV, which implies that the effective Dirac mass M^* of the nucleon in the nuclear medium would become as small as 340 MeV. Furthermore, this analysis yields quite a strong momentum dependence of the scalar and vector components of the self-energy Σ. Because of the strong cancellation between the scalar and vector components, the total effect of the π exchange on the single-particle energy

$$\epsilon(k) = \sqrt{M^{*2}(k) + \mathbf{k}^2} - \Sigma^0(k) - M \tag{21}$$

is rather small.

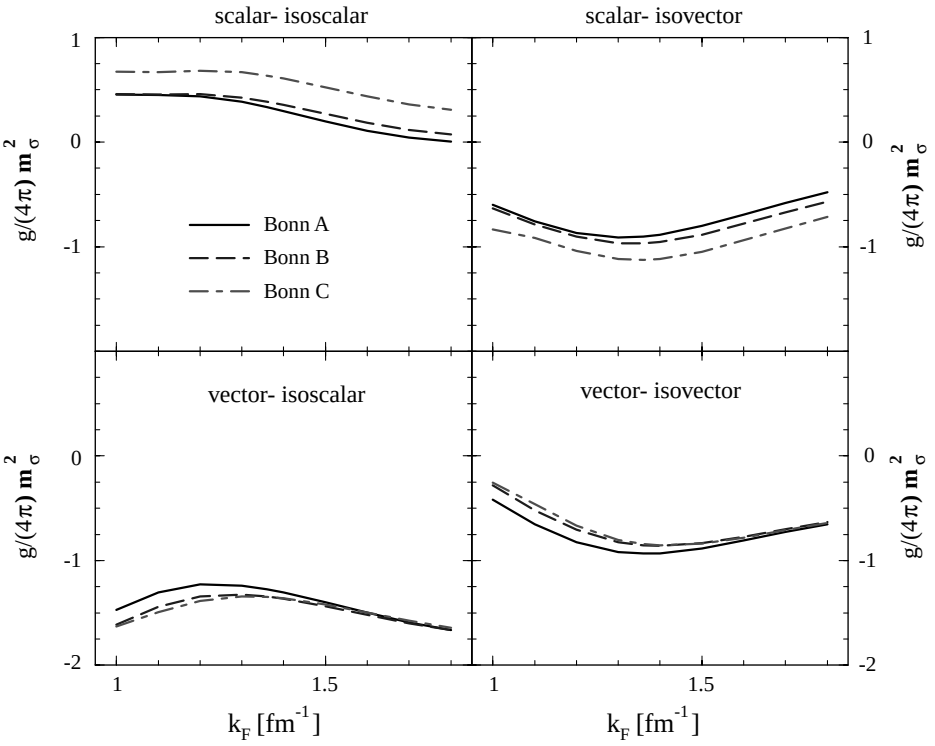

Fig. 6. Coupling constants for the parametrization of ΔG according to eq. (22) for the three versions of the Bonn potentials. These coupling constants are presented as a function of the Fermi momentum $k_{\rm F}$ of symmetric nuclear matter. The values of the constants are multiplied with the square of the mass of the σ-meson ($m_\sigma = 550$ MeV) and divided by 4π to obtain dimensionless quantities of the order 1.

Calculating the self-energy directly from the π exchange part in the Bonn B potential leads to quite different results as one can see from the right-hand side of fig. 1. In this case, the scalar part of the self-energy has the opposite sign, it is small (of the order of 15 MeV) and exhibits a very weak momentum dependence. The single-particle energy for the nucleons in nuclear matter, calculated according to (21), is identical to the one derived from the projection method, the Dirac structure of the self-energy, however, is completely different. The reason for this difference is well known: The matrix elements of the π exchange potential, calculated for the positive-energy solution of the Dirac equation for the nucleon, are independent of assuming either pseudo-scalar or pseudo-vector coupling for the πN vertex. The coupling between the positive- and negative-energy spinors, however, is quite different. The projection method analyses the matrix elements of the positive-energy solutions in term of a pseudo-scalar operator, leading to large components of the self-energy. Assuming a pseudo-vector coupling for the pion, as it is done in the original NN interaction, V yields much smaller components in the self-energy. This feature is known for a long time (see, *e.g.*, [15]). It is the origin of the strong momentum dependence of the self-energy observed in [26], where the projection method has been applied. The problem might be cured by analysing the interaction in terms of the pseudo-scalar operator, transforming the pseudo-scalar component into a corresponding pseudo-vector term and evaluate the self-energy with this pseudo-vector part (see, *e.g.*, [25,27]).

Such ambiguities of the projection method, however, not only arise for the π exchange part. Another example is of course the exchange of the pseudo-scalar–isoscalar meson, the η-meson. Because of its weaker coupling constant and its higher mass, it does not play such a significant role as the π exchange. Nevertheless, also in this case the differences between the self-energy components derived via the projection method and from the direct evaluation are non-negligible as one can see from fig. 2.

The ρ exchange contribution to V is considered as a last example for such a comparison between the self-energies calculated via the projection method and the direct evaluation of the meson exchange term. Results are displayed in fig. 3. The original ρ exchange term of the Bonn B potential contains a vector coupling but also a strong tensor coupling (Pauli-coupling) term in the Lagrangian for the ρN interaction. Also these two different coupling modes cannot be resolved by means of the projection method. The difference between the direct evaluation of the self-energy and the use of the projection formalism are remarkable also in this case.

Therefore, in order to minimise the ambiguities in the projection method, we suggest to split the effective interaction G according to eq. (20) into the bare interaction V and the correction term ΔG, representing the corrections which are due to the correlations. The Dirac

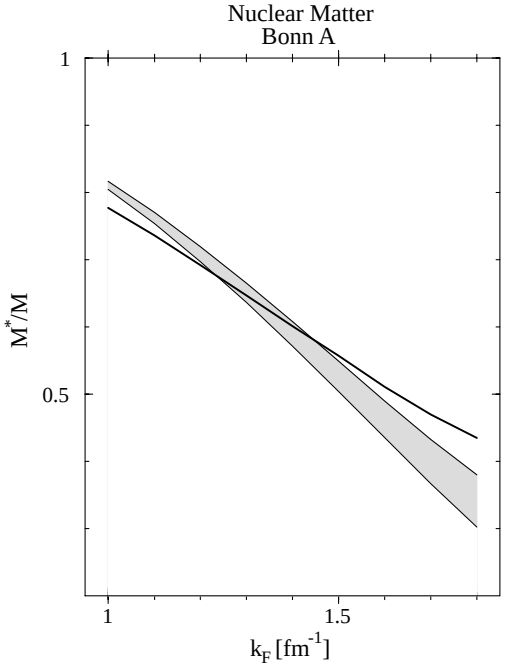

Fig. 7. Effective mass M^* characterising the Dirac spinor of the nucleon in nuclear matter as a function of the Fermi momentum $k_{\rm F}$. The range of the momentum-dependent masses derived from the projection method are indicated by the shaded area, the result obtained from an analysis of the single-particle spectrum is displayed by the solid line. The data have been derived from the Bonn A potential.

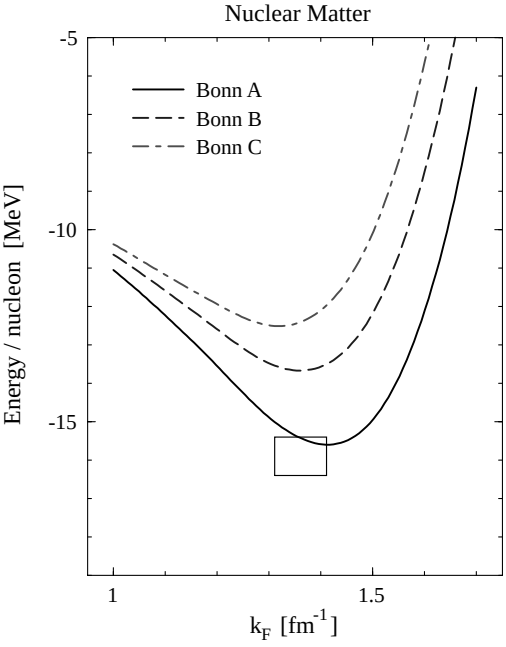

Fig. 8. Calculated binding energy of nuclear matter as a function of the Fermi momentum $k_{\rm F}$ using the three different versions of the Bonn potential.

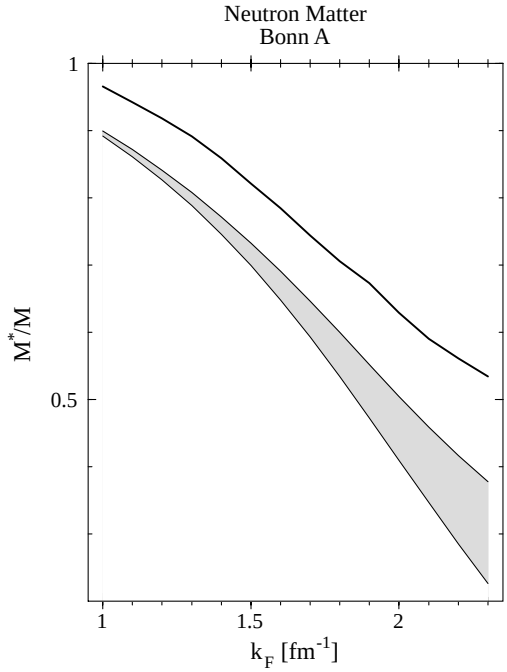

Fig. 9. Effective mass M^* characterising the Dirac spinor of the nucleon in neutron matter. For further details, see fig. 7.

structure of the bare interaction is directly known, and the projection method has to be applied to the analysis of ΔG only. This scheme is advantageous also from another point of view: One must keep in mind that the matrix elements of V and G in the helicity basis are dominated by the one-pion-exchange contribution. Note that the ratio of coupling constant and mass of the meson, g_α^2/m_α^2, which is a measure for the importance of the various meson exchange contribution is about a factor 25 larger for $\alpha = \pi$ than for the σ or ω-meson. Analysing ΔG this dominating π exchange contribution, including the momentum dependencies, which are related to the form-factors for the πN vertex, are removed, which stabilises the numerical analysis significantly.

Results for the nucleon self-energy calculated again for symmetric nuclear matter at a Fermi momentum $k_{\rm F} = 1.36$ fm^{-1} are displayed in fig. 4. The contribution of the bare NN interaction V to the scalar and vector part of the self-energy (see dashed lines in both parts of the figure) yields rather similar values ranging from -420 MeV to -400 MeV for $k_1/k_{\rm F}$ between 0 and 1. This implies that the single-particle energies tend to be positive and one obtains no binding energy for nuclear matter. This is in line with the non-relativistic studies mentioned in the introduction: Using realistic NN interactions one does not obtain any binding energy if correlations beyond the Hartree-Fock approach are ignored.

Adding the contributions of ΔG to the various components of the self-energy, one obtains the results displayed by the solid lines in fig. 4. The correlation effects contained in ΔG reduce the absolute value for the scalar as well as the vector component of the self-energy. One could argue

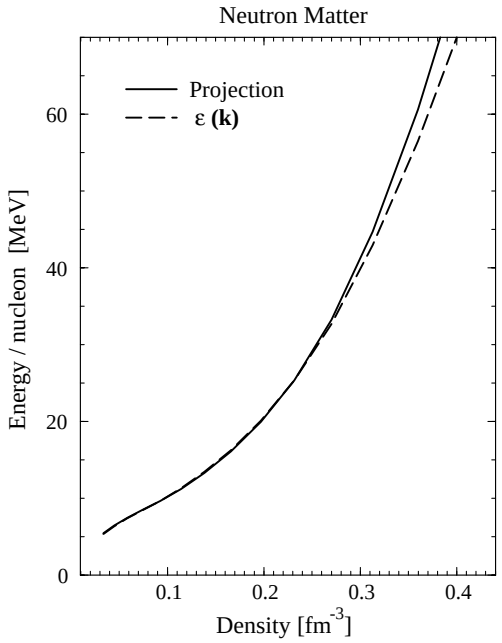

Fig. 10. Calculated binding energy of neutron matter as a function of the density using the Bonn A potential. The results derived from the projection method are compared to those derived from the analysis of $\epsilon(k)$.

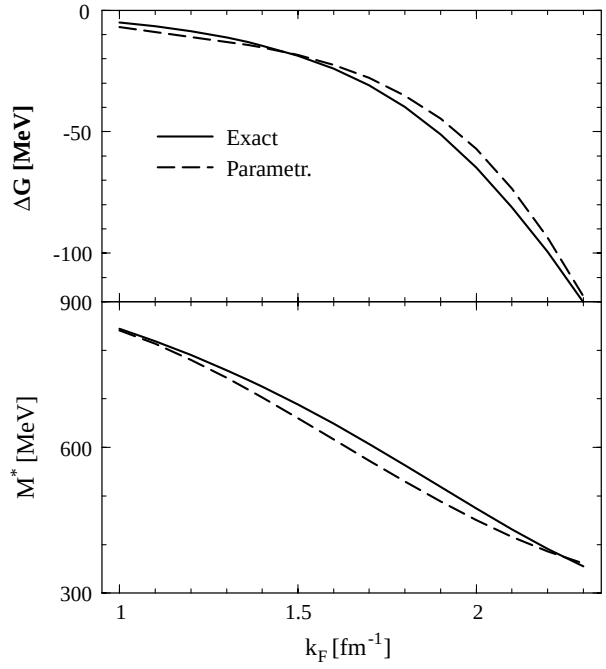

Fig. 11. Contribution of ΔG to the energy of neutron matter and the effective mass M^*, calculated at $k = k_{\rm F}$, as a function of $k_{\rm F}$. The results of the direct calculation (solid lines) are compared to the prediction of the parametrization of ΔG according to eq. (22) assuming that this parametrization is independent of the isospin asymmetry.

that the short-range correlations lead to a reduction of the wave function at small relative distances. This quenching of the relative wave function reduces the effects of the σ exchange, which is the dominating contribution to $\Sigma^{\rm s}$, as well as the ω exchange, which is the driving term in Σ^0. Since the ω exchange is of shorter range than the σ exchange, the mass of the ω-meson ($m_\omega = 783$ MeV) is larger than the one of the σ ($m_\sigma = 550$ MeV), the quenching of the short-range components in the relative wave function is more important for the ω than for the σ exchange. This explains that the absolute value of Σ^0 is reduced stronger by ΔG than the absolute value of $\Sigma^{\rm s}$. This difference on the effect of correlations on $\Sigma^{\rm s}$ and Σ^0 leads to attractive single-particle energies and binding energy of nuclear matter.

It is worth noting that the inclusion of ΔG leads to a constant shift in the components of the self-energy, which is almost independent of the momentum of the nucleon. This very weak momentum dependence of the ΔG effects can also be seen in fig. 5, where the antisymmetrized amplitudes $\Gamma^i_{\rm A}$ (see eq. (15)) are displayed. These amplitudes are derived from the analysis of ΔG. Also these interaction amplitudes are almost independent of the relative momentum q. Deviations from the constant value are observed only for $q \to k_{\rm F}$, *i.e.* for momenta for which the energy denominator in the Bethe-Goldstone equation approaches the pole.

This observation suggests to parametrize the component ΔG in terms of the exchange of effective mesons with infinite mass. To put it in different words: The correlation effects contained in ΔG are described in terms of an effective interaction with zero range. If we focus the attention on the scalar and vector interaction amplitudes only, at each density, 4 coupling constants are required to parametrize ΔG in the form

$$\Delta G = [g_{\rm s,s}\mathbb{1}\mathbb{1} + g_{\rm v,s}\gamma^\mu\gamma_\mu] + [g_{\rm s,v}\mathbb{1}\mathbb{1} + g_{\rm v,v}\gamma^\mu\gamma_\mu]\,\tau\cdot\tau\,. \quad (22)$$

The results for these effective coupling constants are displayed in fig. 6.

The value of these coupling constants indicate that ΔG is in general weaker than V but yields a non-negligible correction to the bare interaction V. The parameters are rather similar for the three versions of the Bonn potential (Bonn A, B and C as defined in table A.2 of [11]). The absolute values are typically larger for Bonn C than for the other two reflecting the fact that this potential yields slightly stronger correlation effects. The density dependence of these parameters is weak but non-negligible. The largest absolute values are observed for vector-isoscalar pseudo meson ($g_{\rm v,s}$). This reflects the fact, which we mentioned already above, that correlations yield a suppression of the ω-meson exchange in particular.

Results for the properties of symmetric nuclear matter are displayed in figs. 7 and 8. Figure 7 displays the density dependence of the effective Dirac mass $M^*(k)$ defined in eq. (4). The shaded area at each Fermi momentum $k_{\rm F}$ indicates the range of values for $M^*(k)$, which is obtained at the corresponding density using the projection method. For a comparison, we also present the effective mass derived from the momentum dependence of

the single-particle energy as proposed by Brockmann and Machleidt [17]. The analysis of the single-particle energy, which is much simpler than the projection scheme, yields rather similar results for symmetric nuclear matter. Therefore also the calculated binding energies, derived from the projection scheme (see fig. 8), are very close to those obtained in [17].

Significant differences between these two methods, however, can be observed in the case of isospin asymmetric matter, like, *e.g.*, pure neutron matter. The effective masses obtained from the single-particle spectrum are significantly larger than the range of $M^*(k)$, which are derived at the same density from the projection method (see fig. 9). This confirms the observation of Ulrych *et al.* [21] who studied a parametrization of G published by Boersma and Malfliet [25]. This demonstrates that the analysis of the the single-particle energy is not a reliable tool to determine the Dirac structure of the self-energy: accidentally it works well for symmetric nuclear matter for the NN interactions considered, it fails, however, for asymmetric nuclear matter.

How does this affect the calculated binding energies? As an example, results for the binding energy of pure neutron matter calculated for the Bonn A potential are presented in fig. 10. The two methods yield almost identical results at low densities. At densities around 0.4 fm^{-3}, which corresponds 2.5 times the saturation density of nuclear matter, the evaluation based on the simple analysis of $\epsilon(k)$ underestimates the energy by 10%.

Finally, we add a remark on the asymmetry dependence of the parametrization of ΔG according to (22). If one employs the density-dependent parameters for ΔG derived in symmetric nuclear matter as displayed in fig. 6, assuming that the parametrization of ΔG is independent of the isospin asymmetry, also for pure neutron matter one obtains effective masses (M^* at $k = k_{\rm F}$) and contributions of ΔG to the energy per nucleon in neutron matter as presented by the dashed lines in fig. 11. These results are very close to the corresponding values derived from a direct determination of ΔG in neutron matter. This indicates that the underlying assumption, the parametrization of ΔG is independent of the isospin asymmetry, is quite reasonable. Therefore, one may use the parametrization of ΔG as displayed in fig. 6 for Dirac-Brueckner-Hartree-Fock studies of finite nuclei with $N = Z$ as well as $N \neq Z$.

4 Conclusions

A method is presented which determines the Dirac structure of the Brueckner G-matrix from its matrix elements between positive energy spinors only. The usual projection method is model dependent as it depends on the choice of the covariant operators. This model dependence is demonstrated for various meson exchange terms. In order to minimise the model dependence of the projection method G is split into the bare interaction V and the correction ΔG reflecting the effects of NN correlations. Since the Dirac structure of V is known, the projection methods is applied to the correction ΔG only. This analysis allows an explicit study of the correlation effects on the Dirac structure of the nucleon self-energy. A simple parametrization of ΔG and its density dependence is presented in terms of the exchange of pseudo-mesons with infinite mass. This parametrization could be useful for the study of finite nuclei. It is shown that the simple method, which determines the Dirac structure of the self-energy from the momentum dependence of the single-particle energy yields fairly good results for symmetric nuclear matter but fails for asymmetric matter.

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